

AN002: Pound-Drever-Hall Locking with the FSC

This Application Note details how to set up a laser-locking apparatus for frequency stabilisation and linewidth reduction using the Pound-Drever-Hall (PDH) technique, which can be used to perform high-bandwidth locking suitable for many applications requiring a stable laser frequency. The Note focusses on practical implementation of working with a high-finesse cavity and locking the laser at an arbitrary frequency.

- 1. Introduction to the technique
- 2. Apparatus design
- 3. Cavity alignment
- 4. Error signal creation and optimisation
- 5. Configuration of the FSC
- 6. Modification for locking at arbitrary frequencies

1. Introduction

For a detailed introduction to the background and theory of the PDH technique, see the excellent paper by E.D. Black (*An introduction to Pound-Drever-Hall laser frequency stabilization*, Am. J. Phys. **69** (1) 79-87, 2001), dx.doi.org/10.1119/1.1286663. The article describes the underlying physics and motivation of the technique, but is not required reading for this Note.

To stabilise the frequency of a laser, the laser frequency must be measured so that feedback can be applied to the laser controller to oppose any fluctuations. Typically the measurement is achieved by comparing the frequency to some fixed reference such as an atomic absorption line or optical cavity resonance. An *error signal* quantifies how far the actual frequency is from the reference (the *offset* or *detuning*).

An ideal error signal (Figure 1) is linear in the offset from the reference value, but in practice saturates at some value (as limited by power supply voltage, for example). The sign of the signal describes whether the measured value is *above* or *below* the reference, and the magnitude quantifies *by how much*. Laser controllers such as the MOGLabs DLC series provide a servo which takes this *error signal* and generates a *control signal* which adjusts the laser frequency via *actuators*, typically the laser diode current and a piezo transducer which affects the laser cavity length. The control signal effectively adjusts the laser frequency to reduce the error signal towards zero. This closed feedback loop suppresses fluctuations and keeps the laser tuned to the frequency required for experimentation.



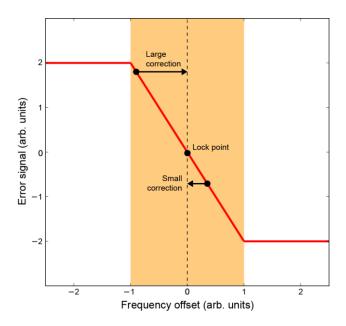


Figure 1: Schematic of an ideal error signal. The sign and magnitude describe the correction required to drive the measured frequency to match the reference. Although the signal saturates, it remains monotonic.

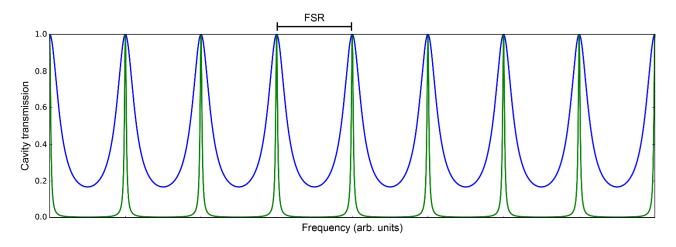


Figure 2: Theoretical transmission for an ideal cavity. The cavity reflects most of the incident light, unless the frequency matches one of the cavity modes. Higher reflectivity mirrors increase the cavity finesse and result in sharper fringes (green) as compared to lower finesse (blue).

The cavity absorption/transmission does not have the necessary properties to be directly used as an error signal as it is symmetric with respect to detuning (Figure 3A). However, the cavity imparts a *phase shift* that depends on whether the frequency is above or below the nearest resonance (Figure 3B). To account for this phase shift, we introduce the complex-valued reflection coefficient, $F(\omega) = E_{\rm reflected}/E_{\rm incident}$. The imaginary part of this quantity is strongly dependent on the phase and resembles a dispersion relation (Figure 3C), which could be used to generate an error signal. The question of how to measure this quantity motivated the development of the PDH technique.

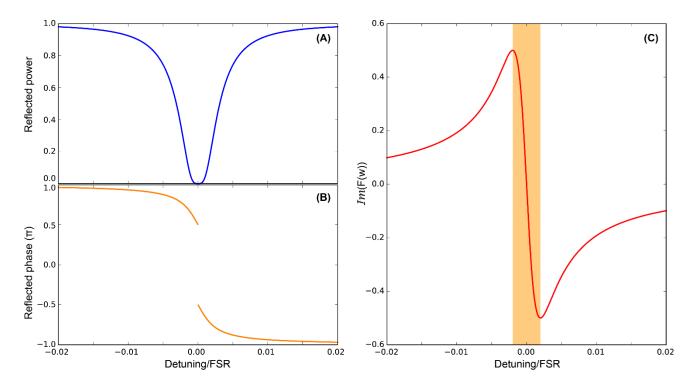


Figure 3: Reflected beam power (A) and induced cavity phase shift (B) near resonance for a lossless symmetric cavity. Note the phase discontinuity on resonance, where the beam is entirely transmitted by the cavity. The imaginary part of the complex reflection coefficient resembles as dispersion profile and the central region (shaded) approximates an ideal error signal.

The key insight which underpins the PDH technique is that modulating the incident beam at a frequency much greater than the cavity linewidth will generate sidebands that are off-resonant with the cavity. These sidebands are promptly reflected from the cavity, and interfere with the near-resonant carrier to form a beatnote. This beatnote contains the phase-shift imparted upon the carrier, which can be recovered by signal processing.

Mixing the beatnote with a local oscillator at the modulation frequency recovers one quadrature of the complex reflection coefficient. An appropriate choice of the local oscillator phase recovers the imaginary part, which can then be used as an error signal (Figure 4). The slope depends on the finesse of the cavity, and can be used as the basis of a very sensitive high-bandwidth servo using cavities of very high finesse, that can even be used for reducing the linewidth of a laser below 1 Hz. Note that while three features are observed corresponding to the different frequency components resonating with the cavity, the central feature is used for locking as it has the steepest slope.

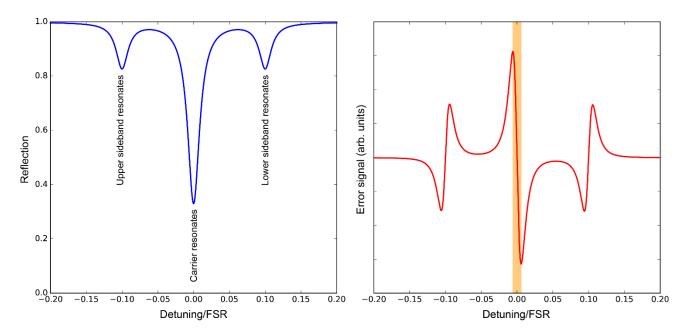


Figure 4: Idealised PDH signal modulating at $f_{PDH}=0.1f_{FSR}$. Modulating the incident laser at a frequency greater than the cavity linewidth yields an absorption profile (left) which shows dips corresponding to each frequency component individually resonating with the cavity. The reflection contains the interference of these frequency components, so when one component experiences a phase-shift from the cavity, the beatnote between the components changes (right), yielding a sharp error signal centred on each resonance.

2. Equipment

The following components are used in this Application Note. We recommend MOGLabs products and describe their specific use in this note, but the techniques described are applicable more generally.

- MOGLabs CEL002 cateye laser
- MOGLabs DLC202 low-noise laser controller
- MOGLabs FSC100 fast feedback servo
- Stable Laser Systems cavity, finesse 20,000 and FSR 1.7GHz, in vacuum housing
- EOSPACE low-Vπ phase modulator (EOM)
- EOTech Tornos series 30dB Faraday optical isolator

The following components are used for alignment and error-signal generation.

- Simple CCD/CMOS camera (e.g. Thorlabs DCC1545m or PointGrey Blackfly)
- High-bandwidth photodetector (e.g. Thorlabs PDA150A)
- Rf mixer (e.g. Minicircuits ZAD-3+, ZFM-2+)
- Low-pass filter (e.g. Minicircuits SLP-5)
- DC-block (e.g. Minicircuits BLK-18)
- Dual-channel function generator (e.g. Rigol DG4062)

The camera is only used for mode identification (described in section 4) so the required functionality is minimal; although the ability to adjust exposure time and gain is convenient. The photodetector requires a bandwidth much greater than the modulation frequency (e.g. 150MHz for modulation at 20MHz) and rf components should be chosen similarly, with bandwidths of order 10 times the desired servo bandwidth.

The PDH technique requires frequency sidebands imposed on the laser carrier. This Note describes using an EOM, but in principle any technique for sideband generation could be used, provided the modulation frequency is significantly greater than the cavity linewidth (see section 5). The dual-channel function generator provides a convenient way to adjust the relative phase between two identical sine waves for error signal optimisation, but in principle a variable phase delay component could be used instead.

The layout of the apparatus is shown in Figure 5. It is strongly recommended to couple the laser to the cavity using an optical fiber, to ensure a clean Gaussian mode as well as reliable long-term pointing stability. Polarisation-maintaining fibers with correctly aligned polarisation axis are necessary to prevent error signal drift and lock instability. Alignment of the cavity is achieved using the camera as described in section 4, and the signal processing required to generate the error signal is detailed in section 5.

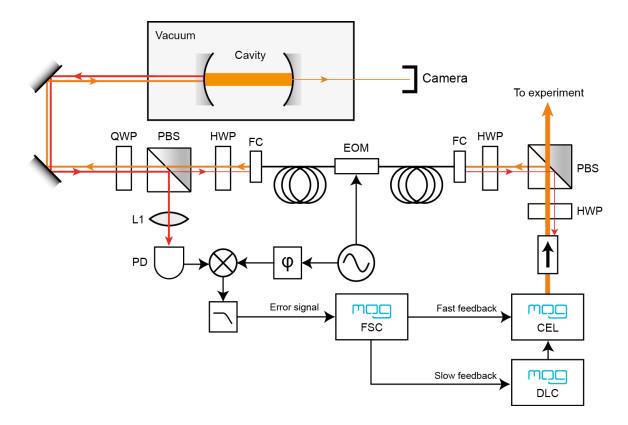


Figure 5: Simplified apparatus diagram for the PDH lock. Some laser light is picked off by a polarising beamsplitter (PBS) and fiber-coupled (FC) into an EOM. The beam is out-coupled and passes through a polarising beamsplitter (PBS) into the vacuum chamber containing the high-finesse cavity. The weak transmission is observed with a camera. The intensity of the reflected beam (red) is measured on the photodiode (PD) and mixed with a phase-shifted copy of the rf driving the EOM. The result is low-pass filtered to produce an error signal which is processed by a fast servo system (FSC) and fed back to the laser (CEL) and its controller (DLC).

Note that the back-reflection from a well-aligned cavity will propagate back through the optical fiber to the source laser, so strong isolation is required (60dB isolation is recommended). Even a small amount of light reflected from the cavity passing through the PBS cubes (due to imperfect extinction) can destabilise the laser mode. Optical isolation can also be improved by introducing a neutral density filter (NDF) between the first PBS and fiber coupler, as only ~100uW of incident optical power are required once the cavity is aligned.

3. Alignment considerations

The process to align the cavity is similar to any Fabry-Perot interferometer, which requires that the laser beam be well mode-matched to the cavity. For low finesse cavities, this is easily achieved by optimising the cavity transmission on a photodiode. Alignment is more complicated for high-finesse cavities because:

- 1. The cavity is typically under vacuum, to prevent the high intracavity power heating the intracavity medium and destabilising the cavity by applying pressure to the mirrors. However, the chamber limits optical access and the vacuum windows create additional reflections.
- 2. The cavity transmission is extremely low. The approximate expected off-resonant transmission for large finesse \mathcal{F} is $T \sim (2\mathcal{F}/\pi)^{-2}$, which for $\mathcal{F} = 20,000$ and 1mW of incident light gives picowatts of transmission.
- 3. The free-spectral range (FSR) is large and cavity linewidth small. Typically modes are identified by scanning the laser across the FSR of the cavity and observing the transmission peaks. However, the ratio of linewidth to FSR is $\sim 1/\mathcal{F}$, so resonance is crossed very rapidly and requires high photodetection bandwidth to see any signal, combined with high gain to compensate the low transmission.
- 4. The cavity supports high-order modes, which can couple strongly and make it difficult to identify the desired fundamental mode. For example, a slight horizontal misalignment could couple more strongly to the TEM01 mode than the required TEM00 mode. Typically identification of the fundamental cavity mode is achieved by tweaking the alignment to find the strongest mode coupling which shows no other mode excitation. With a high finesse cavity, the signal is very weak and sharp so alignment is nontrivial.

4. Rough cavity alignment

It is important to maximise the coupling into the fundamental cavity mode to generate a strong error signal. The process to achieve this is threefold: firstly to roughly align the cavity and reduce the order of the excited modes, secondly to identify the fundamental mode and bring the laser to resonance, then finally to directly improve the coupling.

First, collimate the incident beam, for example using either a shear-plate interferometer or measuring the profile over extended distances.

Mode-matching the beam with the cavity is achieved by matching the Gaussian beam curvature with the curvature of the mirrors. Typically the manufacturer will use measured radius of curvature for the two mirrors to specify the calculated beam waists at the positions of the two mirrors (w_1, w_2) as well as the associated focal waist (w_0) . For example, a plano-concave cavity requires the beam be focussed on the

planar mirror, with beam waist set by the radius of the concave mirror. Typically the required beam waist is $\sim 200 \mu m$. Use a beam profiler to measure the beam waist at equivalent position to the planar mirror (or mid-point of a confocal cavity) and an appropriate telescope configuration to achieve the desired waist.

The initial alignment can be achieved by looking at the back-reflection off the cavity. The high reflectivity of the cavity mirrors means the back-reflection will be quite strong. Reflection from a concave mirror will be divergent whereas the back-reflection from the vacuum windows will be collimated. A distinction should be drawn here between the reflection from the front surface of the cavity mirror, and the "cavity reflection" which enters the cavity and is subsequently reflected. However, the back-reflection from the outside surface is bright and easily used for alignment. For this reason, if using a plano-concave cavity, it is recommended to couple in through the concave cavity mirror rather than the planar mirror because then the strong (~99.9%) back-reflection is divergent.

To achieve finer alignment, we suggest using a sensitive CMOS or CCD camera, such as the Thorlabs DCC1545m or PointGrey BlackFly, to image the transmitted cavity modes directly. The sensor is extremely sensitive at low light levels when operated without a lens, and the spatial information provided by a camera greatly simplifies the alignment procedure. It may be desirable to place a filter in front of the camera to block ambient light where possible.

Position the camera facing the output window of the cavity. Slowly scanning the laser frequency should reveal multiple patterns on the camera as different transverse cavity modes come into resonance at different frequencies (Figure 6). Note that because of the high intracavity power, the incident power should be at most ~1mW to prevent damaging the cavity mirrors. The incident optical power will be further reduced to microwatts during optimisation of the error signal to prevent destabilisation of the error signal by heating effects.

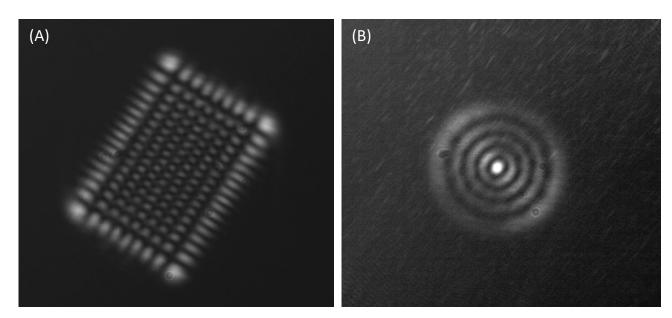


Figure 6: Examples of high-order modes transmitted by a misaligned cavity, showing that both Hermite-Gauss (A) and Laguerre-Gauss (B) modes can be observed by becoming resonant at different laser frequencies.

Observe the shapes of the transmitted modes as the laser frequency is scanned. When the cavity is strongly misaligned, the horizontal and vertical axes are non-degenerate and you will excite high-order rectangular Hermite-Gauss (TEM) modes in the cavity. As the alignment is improved, circularly symmetric Laguerre-Gauss (LG) modes begin to couple more strongly. Hence the spatial structure of the transmitted light yields information about the cavity alignment.

For rough alignment, scan the laser frequency over a range greater than the FSR of the cavity, and make the exposure time of the camera longer than the scan period of the laser. For the MOGLabs DLC this is typically 20ms. The camera will then show an integrated image (average) of all modes supported by the cavity. The structure of this averaged pattern then indicates how alignment needs to be adjusted to improve coupling into the desired fundamental mode (Figure 7).

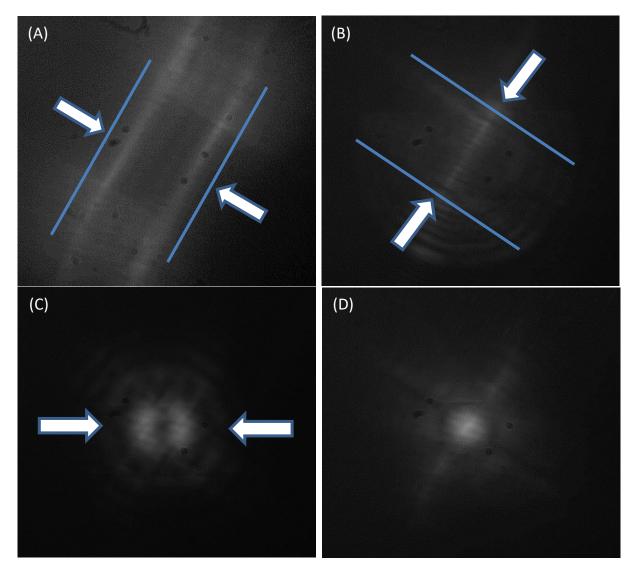


Figure 7: Demonstration of cavity alignment procedure using a wide frequency scan. Initially the "average" mode has a rectangular shape (A). Improving horizontal alignment collapses the rectangle (B) and begins to weakly couple circular modes. Subsequent improvement of vertical alignment results in LG mode shapes (C), although the central intensity minimum zero implies the TEM01 mode is coupled strongly. Tweaking alignment further maximises the coupling into a Gaussian mode (D).

Ensure that the intensity in the centre of the pattern is maximised, to avoid optimising for low-order modes such as TEM01 and LG01 which have zeros in the centre. Decrease the scan range of the laser to ~100MHz and manually tune the frequency until you see a Gaussian transmitted beam profile (Figure 8). This is the fundamental mode of the cavity, which provides the strongest locking signal. Note that a high finesse cavity can have a linewidth less than the free-running linewidth of the laser so it is very easy to sweep past. However, the mode should couple very strongly and may saturate the camera, which aids identification.

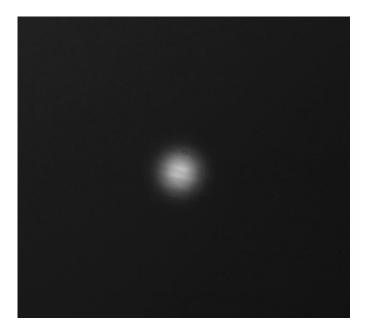


Figure 8: Fundamental mode of the cavity observed by reducing the span of the laser frequency sweep and manually adjusting the centre frequency to bring the cavity into resonance.

Finally, once the mode is identified, make minor adjustments to the alignment to increase the transmitted power. A high bandwidth photodetector could be used, placed at the cavity output, but it is sufficient to maximise the intensity recorded by the camera. Final adjustments will be made once an error signal is obtained. If your camera software provides a 1D lineout of the image, this is a convenient way to maximise the transmission.

5. Generating the error signal

Once the cavity is aligned and the fundamental mode identified, it is possible to generate an error signal. The key insight of the PDH technique is that the error signal is derived from variations in the phase shift for light in the cavity at frequencies above and below resonance. Adding frequency sidebands to the laser beam creates frequency components of the light field above and below resonance, and interference between those components creates a beatnote which can be measured with a photodetector. Demodulation of the beatnote using the modulation frequency reference creates the error signal.

Regardless of the technique chosen to generate the sidebands, the key parameters are the modulation frequency and the modulation depth. The PDH technique is not particularly sensitive to either, provided the modulation frequency is significantly larger than the cavity linewidth but within the detection bandwidth of the photodetector (PD). The parameters can therefore be chosen to match available equipment.

Note that the chosen modulation technique may have unintended consequences that affect the resulting error signal. For example, an AOM induces a combination of AM and FM. An EOM induces polarisation rotation that is converted into AM by the PBS used to isolate the cavity reflection. Residual AM is undesirable as it results in a DC-offset in the error signal which is proportional to the optical power, so drift in optical power will result cause the laser to unlock. To minimise this drift, ensure good polarisation purity into the EOM and keep the modulation depth is small (much less than quarter-wave). Once an error signal is obtained, the half-wave plate can be adjusted to eliminate the residual DC offset. In particular, when using polarisation-maintaining fibers (e.g. PANDA fiber) the input polarisation must be carefully aligned to either the fast or slow fiber axis to prevent strain-induced ellipticity. This ellipticity drifts sensitively with temperature and affects lock stability.

Having identified the fundamental mode, a photodetector should be used to measure the reflections from the cavity. With no modulation and as the laser frequency is swept across resonance, a dip will be seen in the reflected power corresponding to resonance. Make small adjustments to the alignment to increase its depth. It is recommended that the reflected light be collected with a focusing lens in front of the PD.

Apply modulation to the laser, for example using an AOM or EOM, to generate sidebands. The error signal is obtained by demodulating the PD signal with a phase-shifted copy of the driving signal on a mixer (Figure 9). A dual-channel function generator is convenient for this during optimisation, but can later be replaced by appropriate fixed-value components of lower cost. The low-pass filter on the mixer output must have a cut-off frequency less than the modulation frequency to eliminate the higher-order terms, but high enough to not limit the bandwidth of the lock. For example, good starting values are to modulate at 20MHz and low-pass filter at 5MHz.

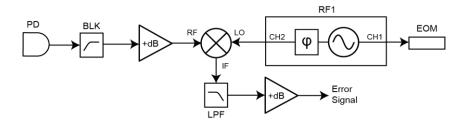


Figure 9: Demodulation scheme to derive an error signal with the PDH scheme. RF1 is used to drive the EOM and demodulate the PD signal. A DC-block (BLK) may be required mixer depending on the amplifier and mixer used. The low-pass (LPF) filter has a cut-off well below the modulation frequency to eliminate higher-order terms that would otherwise contaminate the error signal.

¹ Note that when the HWP before the PBS is rotated to give maximum transmission, the resulting AM is at *twice* the driving frequency, whereas when it is set for 50% transmission, the AM occurs at the driving frequency and produces a DC offset in the error-signal.

Depending on your modulation technique, the phase difference should be approximately $\pi/4$, with the exact phase dependent on the components used. For example, when directly modulating the laser current, there is typically a frequency-dependent phase-shift in the modulation electronics which must be compensated by the applied shift. In practice, once a demodulation signal is achieved, the phase can be adjusted until the resulting error signal has the desired shape (Figure 10).

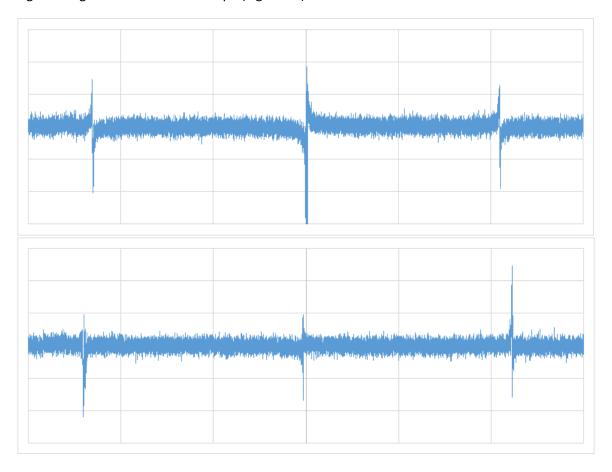


Figure 10: Typical error signals generated from a PDH apparatus. When the demodulation phase is correct the signal is symmetric (top), whereas when the phase is incorrect the sidebands are asymmetric and the magnitude of the central feature is reduced (bottom).

The amplitude of the local oscillator (LO) should also be varied to optimise the signal-to-noise ratio (SNR) of the error signal. Mixers are non-linear devices and their response depends on the power of the LO. If the power is too low the error signal will be weak, but if the power is too high then the noise contribution is increased. Many mixers are designed for +7 dBm LO (0.7 Vpp into 50 ohms), which indeed was optimal for the Minicircuits ZAL-3+ mixer.

The incident optical power should typically be kept small, of order $100\mu W$ for a cavity with finesse of 20,000. More power results in a noisy signal because of heating effects, but reducing the power requires more amplification during signal processing, which increases the electrical noise. Hence there is a balance between optical power and electronic amplification.

6. Locking using the MOGLabs FSC

The narrow linewidth of the cavity resonance means that a high-bandwidth servo controller is required to lock the laser to the PDH error signal. Often the cavity linewidth is less than the free-running linewidth of the laser, meaning that a very fast control loop is required to achieve a stable lock, which has the benefit of then reducing the laser linewidth ("linewidth narrowing"). The MOGLabs *FSC100* is one such servo controller that provides individually-configurable control loops for both fast feedback (laser current) and slow feedback (piezo). This section outlines how to configure an FSC for PDH-locking, and how to integrate with a MOGLabs laser system comprising a CEL laser and DLC controller.

a. Configuring the CEL and DLC for use with FSC

To achieve the necessary lock bandwidth, it is necessary to directly inject current into the laser diode through the headboard. The B1240 headboard is strongly recommended for this purpose, although the B1047 may be appropriate for laser systems that are incompatible with the B1240. There are three jumpers on the B1240 headboard, which must be set to "DC" and "Buf" (Figure 11). Typical factory default settings are "AC" and "Dir", which will not produce the required behaviour. Connect the "FAST" output of the FSC to the SMA input on the laser headboard. It may be desirable to remove the DVI cable when connecting the SMA. Note that the laser current on the DLC may need to be increased slightly afterwards.

Note: accessing the headboard jumpers requires turning the laser head upside down, potentially causing loss of alignment.

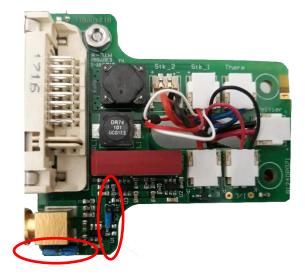


Figure 11: The B1240 headboard with jumper switches identified.

The DLC is used by the FSC as a high-voltage piezo driver and bias current generator. The "SLOW" output should be connected to the "SWEEP/PZT MOD" input and **DIP9** should then be enabled inside the DLC. Ensure that DIP13 and DIP14 are off. The **FREQUENCY** on the DLC should be set to 0.0 and the **SPAN** set to maximum. It is important that the SPAN is subsequently not adjusted on the DLC; the scan should be entirely controlled through the FSC front-panel.

b. Configuring the FSC for PDH

The FSC is a flexible controller suitable for a wide range of applications; the following procedure provides a specific method and approximate starting values suitable for PDH locking. Note that actual values will depend on both the optical cavity being used, and the magnitude of the error-signal produced. Typically the PDH error signal should be ~10mVpp, although higher gain can be used for smaller signals.

- Set "SWEEP" to "INT", then adjust "SPAN" and "FREQ OFFSET" until the PDH resonance is observed.
- Connect the PDH error signal to the "A IN" input. Adjust the demodulation phase and waveplates until the error signal has the expected shape and the DC level is zero.
- Set "INPUT" to "Δ" and "CHB" to zero.
- Set the "MON1" output to "FAST ERR" and adjust the "ERR OFFSET" knob until the DC level output on "MON1" is zero. The lock is especially sensitive to drift in the DC level of the error signal, and the "ERR OFFSET" may need to be periodically adjusted.
- Set "FAST" to "SCAN+P" and observe the PDH resonance. Increase the FAST gain and check the effect of inverting "FAST SIGN". The sign and fast gain are approximately correct when the error signal is "stretched out" as shown in Figure 12.

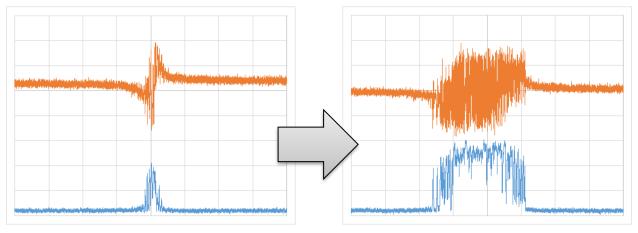


Figure 12: PDH error signal (orange) and cavity transmission (blue) of the cavity resonance (left) is stretched out by using SCAN+P to apply some feedback while scanning (right). When the sign of the error signal is correct, the feedback stretches out the resonant feature in proportion to the fast feedback gain.

- If "FAST SIGN" was inverted, then "ERR OFFSET" must typically be readjusted.
- Set "FAST INT" to 100k, "GAIN LIMIT" to 30, "FAST DIFF" to 250k and "DIFF GAIN" to 12dB.
- Reduce "SPAN" to zero, set "FAST" to "LOCK". Adjust "FREQ OFFSET" to compensate for small DC offsets, until the controller locks to the cavity resonance as observed in the cavity transmission signal.
- Use the diagnostic imaging camera to verify that the laser is locked to the fundamental cavity mode.
- Adjust "FAST GAIN" to improve stability.
- Set "SLOW INT" to 100 and "SLOW GAIN" to mid-range.
- Set "SLOW" to "LOCK". If the laser immediately unlocks, try inverting "SLOW SIGN". It may be necessary to tweak "ERR OFFSET" before engaging the slow lock.
- If engaging the slow lock causes a DC shift in the error signal, make small adjustments to the trimpot next to the "ERR OFFSET" knob until no measureable shift is observed.

c. Lock optimisation

The cavity transmission provides an excellent independent measure of the lock quality. The cavity transmits almost no light when off-resonant, and settles to a steady non-zero DC level when the laser is well-locked to the cavity. It is recommended to use a 50/50 non-polarising beam-splitter on the cavity output to enable it to be recorded both on a high-speed photodiode as well as a camera for diagnostic mode-identification.

To optimise the slow gain, observe the response to acoustic perturbations. Unlike many servo controllers, the slow controller may not oscillate when the gain is too high as the fast servo loop will compensate for it. However, when the gain is too high the laser will become very sensitive to acoustic perturbation. Typically the slow servo only needs to compensate for slow drift so significant gain is not generally required.

Spectrum analysis of the cavity transmission makes clear the effect of adjusting the fast servo gains, although it is generally sufficient to observe the transmission on an oscilloscope. The lock parameters should be optimised to produce the steadiest output at the highest DC level. In particular, PDH locks are substantially improved by adding derivative feedback to the current. For the cavity considered here, "FAST DIFF" of 250k with "DIFF GAIN" of 24dB was optimum (Figure 13).

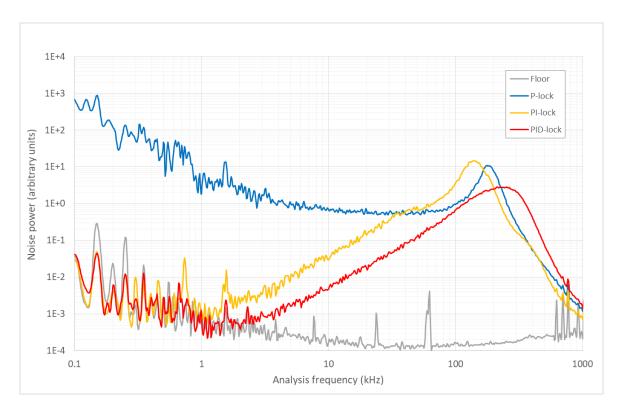


Figure 13: Spectrum analysis of the cavity transmission for a locked laser using SCAN+P (blue), fast lock with FAST DIFF set to OFF (yellow), and final optimised measurement (red).

7. Tips for improving the lock

The following points may be valuable for improving the stability of the PDH lock and linewidth reduction.

- Use a telescope to match the beam-waist with the fundamental cavity mode by matching the radii of curvature of the mirrors, typically specified by the manufacturer. This will make the system considerably less sensitive to beam alignment.
- Use a PBS cube with high extinction ratio (e.g. 10⁵:1 or better) for PDH detection and consider a zero-order quarter-waveplate for improved polarisation purity within the optical cavity.
- Use polarisation-maintaining optical fibers, and carefully align the incident polarisation to either the
 fast or slow fiber axis to minimise strain-induced ellipticity. It may be necessary to use quarter-wave
 and half-wave combinations to counter small amounts of ellipticity. A polarisation analyser is
 extremely helpful in achieving good matching for polarisation stability and therefore error signal
 offset stability.
- Use active laser power stabilisation to ensure the intra-cavity optical power remains constant.
- Place optical isolators on either side of the EOM to prevent back-reflections interfering with laser mode stability a single-stage isolator in front of the laser may be insufficient. Alternatively, a neutral density filter after the EOM can provide some additional isolation.
- Active temperature stabilisation of the cavity will prevent its resonant frequency drifting with time.
 The manufacturer should specify the optimum temperature; that is, the temperature of the zero crossing of the coefficient of thermal expansion, where the cavity becomes first-order insensitive to temperature fluctuations.
- Small adjustments to the "ERR OFFSET" may improve the lock quality, to ensure the electronic zero of the error signal coincides with the central of the cavity resonance.
- The integrators of the servo loops have very high gain at low frequency, making the feedback loop sensitive to power-line pickup noise and ground loops. Take care to isolate the signal path and ensure that ground connections are physically close together.
- Many double-balanced mixers require their RF-input to have a DC component of zero, so a DC block should be used on the photodiode output.

8. Locking to an arbitrary frequency

The technique discussed thus far allows for high-precision laser locking to one of the cavity resonances. However, these are necessarily widely spaced (typically of order GHz) and generally do not coincide with the desired laser frequency (Figure 13). Although some cavities can be frequency-shifted by changing the cavity length (e.g. with piezo, temperature control, or an intracavity optical flat), in general it is preferable to keep the cavity frequency inherently fixed. Instead, the light used to probe the cavity is frequency-shifted to match a cavity resonance. The required frequency shift can be up to half the cavity FSR; for a 1.7 GHz FSR that implies a frequency shift of up to 850 MHz, which is difficult using AOMs but well within the range of EOMs.

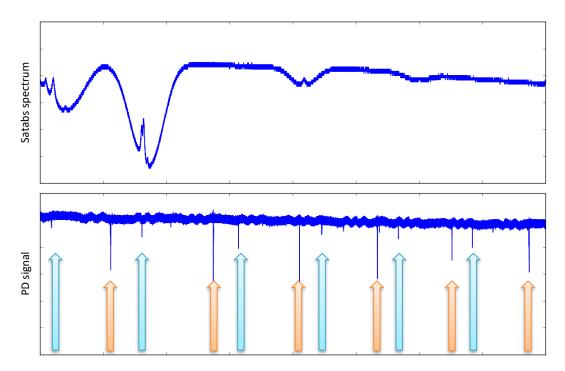


Figure 13: Scanning the laser across the absorption lines of a natural abundance rubidium cell shows the fundamental modes of the cavity (orange). To lock to a specific resonance requires a method to shift either the cavity resonance or the frequency of the light coupled to the cavity. Note the presence of higher order modes (blue) indicates the cavity alignment was imperfect for this measurement.

Once the cavity has been aligned and the cavity modes observed, the "offset" of the desired frequency relative to the closest cavity mode f_{OFS} can be accurately measured. Modulating the EOM at this frequency generates a resonant sideband which then interacts with the cavity. Applying the PDH technique to this sideband by adding additional modulation (f_{PDH}) generates secondary sidebands which produce an error signal that can be used to lock the carrier frequency. Although this approach generates a spectrum of sidebands (Figure 14), only the resonant mode interacts with the cavity and contributes to the error signal.

This result can be achieved by driving the EOM with a combination of the two RF sources using a mixer (Figure 15). An ideal mixer produces frequency components at only the sum and difference frequencies of the inputs, but in this case it is necessary that RF2 leaks into the output. Typically this is achieved by making RF2 strong compared to RF1, and then adjusting the individual powers to optimise the resulting error signal. A single-balanced mixer with LO rejection is superior to a double-balanced mixer in this case. Additional amplification may be required, although be sure to not exceed the maximum input voltage of the EOM.

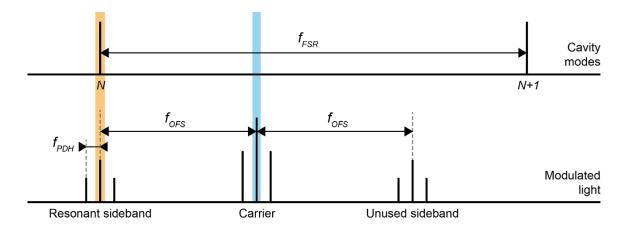


Figure 14: The laser frequency required to perform some experiment (blue) is detuned from the nearest cavity mode (orange) by a substantial fraction of the free spectral range (f_{FSR}). To generate an error signal to lock the laser, the EOM is modulated at two frequencies; f_{OFS} to create a resonant sideband and f_{PDH} to generate the PDH error signal.

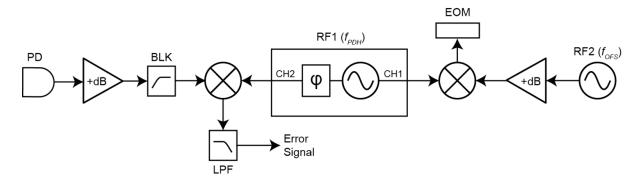


Figure 15: Modulation scheme for PDH with offset. RF2 generates a resonant sideband which interacts with the cavity, allowing the PDH technique to generate an error signal using the dual-channel function generator RF1. Note that the scheme relies on RF2 leaking through the mixer, which requires RF2 to be strong compared to RF1.

To allow for maximum tuneability, RF2 should be capable of generating frequencies over the entire frequency range $4f_{PDH} < f_{OFS} \le f_{FSR}/2$. A high-speed DDS is convenient for this, as it can provide computer-controlled frequency and power control. Note that additional filtering/amplification of the DDS output may be required to produce a clean spectrum as higher-harmonic frequency components can interfere with sideband generation.

In the first instance, drive the EOM directly with RF2 and ensure the resonant sideband can be observed by looking for cavity transmission with the carrier (laser) set to the desired frequency. Introduce the mixer and compensate for losses, then mix in RF1 and check for the error signal. Direct control of the powers of RF1 and RF2 (for example, generating each with a DDS) allows for simplified optimisation of the parameters.

This approach works best when $f_{OFS} \gg f_{PDH}$ so that any undesired sidebands are blocked by the low-pass filter during demodulation of the error signal. Given that $f_{FSR} \gg f_{PDH}$, this will work for most arbitrary choices of frequency.

In the event that the desired carrier frequency is *close* to a cavity mode, two solutions are to either use an AOM to generate the cavity-resonant component for PDH locking, or generate a component resonant with the *next closest* cavity mode by setting RF2 to $f_{FSR} - f_{OFS}$.

To check this, the offset frequency for a given desired frequency f should be estimated from

$$f_{OFS} \approx f - \left| \frac{f}{f_{FSR}} \right| f_{FSR},$$

where $\lfloor x \rfloor$ denotes the floor function. Note that while there is some freedom in choice of f_{PDH} , ultimately the lock bandwidth is limited by the low-pass filter so f_{LPF} should not be reduced too far.